

# WEIGHTED MINMAX ALGORITHM FOR COLOR IMAGE QUANTIZATION

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## Abstract

The maximum intercluster distance and the maximum quantization error that are minimized by the MinMax algorithm are shown to be inappropriate error measures for color image quantization. A fast and effective (improves image quality) method for generalizing activity weighting to any histogram-based color quantization algorithm is presented. A new non-hierarchical color quantization technique called weighted MinMax that is a hybrid between the MinMax and Linde-Buzo-Gray (LBG) algorithms is also described. The weighted MinMax algorithm incorporates activity weighting and seeks to minimize WRMSE, whereby obtaining high quality quantized images with significantly less visual distortion than the MinMax algorithm.

*Key words and phrases:* color image quantization, minimizing maximum intercluster distance, nonhierarchical clustering, image compression.

## 1. INTRODUCTION

In color quantization a truecolor image is irreversibly transformed into a color-mapped image consisting of  $K$  carefully selected representative colors. When Heckbert proposed the color quantization problem in his seminal SIGGRAPH paper [1], most graphic workstations had CRT monitors with an 8-bit frame buffer, capable of displaying only  $K = 2^8 = 256$  colors at a time. With the advent of the Internet and multimedia eras, 24-bit frame buffers have become more commonplace;

however, the same has made the need for efficient ways to represent images more important. Color quantization is a useful method for obtaining high-quality compressed images for storage and subsequent distribution via the World Wide Web. For a survey of some of the more popular color quantization techniques see [2, 3, 4, 5].

The remainder of the paper is organized as follows. Section 2 briefly describes the MinMax algorithm. A fast and effective method for generalizing activity weighting to any histogram-based color quantization algorithm is presented in Section 3. Section 4 presents a new non-hierarchical technique called weighted MinMax that incorporates activity weighing and is a hybrid between the MinMax and Linde-Buzo-Gray (LBG) algorithms. Section 5 shows that the quality of quantized images produced using the weighted MinMax algorithm is superior to the best version of Xiang's MinMax algorithm.

## 2. THE MINMAX ALGORITHM<sup>1</sup>

In 1985, Gonzalez [6] proposed an approximation algorithm that seeks to minimize the maximum intercluster distance. One of the interesting aspects of the MinMax algorithm is that Gonzalez [6] proved it constructs a  $K$ -split whose maximum intercluster distance is less than or equal to two times optimal solution value. The application of Gonzalez's general purpose MinMax clustering algorithm to color image

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<sup>1</sup> This algorithm is commonly referred to as either the MaxMin or MinMax algorithm; this paper shall use the latter.

<b>Input:</b>	$C$	The set of $N$ unique RGB colors in image $I$
	$K$	The numbers of colors to be selected.
		<i>Assumption:</i> $N > K$ .
<b>Output:</b>	$R_K$	Selected representative RGB colors for image $I$
	$S_K$	$K$ -split for $C$
		// <b>Step 1:</b> Initialization
1	$s_0 \leftarrow C = \{c_1, c_2, \dots, c_N\} \subseteq \text{RGB}$	
2	$r_0 \leftarrow c_1$	
		// <b>Step 2:</b> Iterative Creation of New Clusters
3	<b>for</b> $k \leftarrow 1$ <b>upto</b> $K-1$	
4	$\delta \leftarrow \max \{d_s(c_i, r_j) \mid c_i \in s_j \text{ and } 0 \leq j \leq k-1\}$	
5	$c \leftarrow$ one of the colors whose distance to its representative is $\delta$	
6	<b>move</b> $c$ to $s_k$	
7	$r_k \leftarrow c$	
		// <b>Step 3:</b> Reassignment of Colors to New Cluster
8	<b>for each</b> $j \leftarrow 0$ <b>upto</b> $k-1$	
9	<b>for each</b> $c \in s_j$	
10	<b>if</b> $d_s(c, r_k) \leq d_s(c, r_j)$	
11	<b>move</b> $c$ to $s_k$	
12	<b>for each</b> $k \leftarrow 0$ <b>upto</b> $K-1$	
13	$r_k \leftarrow$ centroid of $s_k$	
14	<b>return</b> $R_K$ and $S_K$	

Figure 1: **PSEUDOCODE FOR XIANG’S BEST VERSION OF THE MINMAX ALGORITHM.**  
The shaded lines indicate changes from the original MinMax.

quantization has been studied and enhanced by several researchers [7, 8, 9 and 10].

Pseudocode for the best version of Xiang’s MinMax algorithm [10] is provided in Figure 1. The basic steps of the original MinMax algorithm are as follows. Initially all of the colors in the image are in the same cluster,  $s_0$ , and its representative color,  $r_0$ , is arbitrarily chosen. A new cluster is created in each of the  $K-1$  iterations of the primary **for** loop. A color that is furthest away from its representative color becomes the representative color of the new cluster. Every other color that is closer to the new cluster’s representative color than to its current representative color is moved to the new cluster. Xiang’s modifications to the original MinMax algorithm are summarized below:

- (i) the centroid of each of the  $K$  clusters produced in the **for** loop in lines 3-11 of Figure 1 become the representative colors for each of the  $K$  clusters (lines 12-13), and
- (ii) the distance function is scaled based on their relative contribution to luminance to partially

compensate for the nonuniform nature of the RGB color space:

$$d_s(x, y) = \sqrt{(0.5 \cdot (x.R - y.R))^2 + (x.G - y.G)^2 + (0.25 \cdot (x.B - y.B))^2}.$$

Neither of these modifications invalidates Gonzalez’s theorem that the  $K$ -split produced by the MinMax algorithm has a maximum intercluster distance that is less than or equal to two times optimal solution value.

### 3. SPATIAL ACTIVITY WEIGHTING

The MinMax algorithm produces quantized images with considerable contouring because it treats all colors equally, whereas higher quality quantization techniques use a histogram to weight each color proportional to its frequency. In histogram-based, non-activity weighted algorithms, the importance of a color is based solely upon its frequency. That is, colors that occur more often in the image are given greater precedence. Various methods for weighting the importance of a pixel based on the spatial

activity of the area of the image where the pixel occurs have been incorporated into color quantization algorithms [5, 11, 12, 13, 14, 15, 16, 17]. This section describes a fast and effective method for generalizing activity weighting to any histogram-based color quantization algorithm [5].

The importance of a color is based upon its frequency and the activity level of the locations where the pixels occur by summing the activity weightings of pixels with the same color. The activity level of a pixel is calculated in a manner similar to Balasubramanian et al. [13]. The activity level  $a(x,y)$  of a pixel at row  $x$ , column  $y$  of a truecolor image  $I$  is measured by the magnitude of the luminance gradient of the pixel given by:

$$a(x,y) = |Y(I(x,y)) - Y(I(x-1,y))| + |Y(I(x,y)) - Y(I(x,y+1))|, \quad (1)$$

where  $Y$  is the luminance of the pixel, and assuming  $x > 0$  and  $y < H-1$ . When  $x$  is 0, the first term of Equation 1 is  $|Y(I(x,y)) - Y(I(x+1,y))|$  and when  $y$  is  $H-1$ , the second term of Equation 1 is  $|Y(I(x,y)) - Y(I(x,y-1))|$ . The activity weighting  $w(x,y)$  of a pixel at row  $x$ , column  $y$  is calculated from its activity level using:

$$w(x,y) = \begin{cases} 1/4 & \text{if } a(x,y) = 0 \\ 1/3 & \text{if } a(x,y) = 1 \\ 1/a(x,y) & 1 < a(x,y) < 12 \\ 1/a(x,y)^{1.25} & 12 \leq a(x,y) \leq 16 \\ 1/16^{1.25} & \text{if } a(x,y) > 16 \end{cases} \quad (2)$$

The author conducted a considerable amount of experimentation with various exponents and cut-off values to derive Equation 2. When the activity level is extremely low (0 or 1) the pixel is considered to be part of a flat region; therefore, it is considered less important than when its activity level is low (2). When the activity level is greater than 16, the pixel is considered to be part of an edge. Thus to prevent the blurring of edges, the activity weighting of a pixel is never less than  $1/16^{1.25}$ .

The weighted histogram  $H_w$  of a truecolor image  $I$  is defined as a total function:

$$H_w: \text{RGB} \rightarrow \mathbb{R} \quad (3)$$

where  $H_w(c)$  is the total activity weighting of the pixels in  $I$  with color  $c$ . By simply substituting the weighted histogram for the histogram, any histogram-based algorithm becomes activity weighted. Thus, a non-activity weighted, histogram-based algorithm may be viewed as an activity weighted, histogram-based algorithm in which all pixels are equally weighted as one. Activity weighting in this manner has been shown to be a fast and effective way to enhance the quality of images produced by histogram-based color image quantization techniques [5].

#### 4. THE WEIGHTED MINMAX ALGORITHM

The weighted MinMax algorithm uses the weighted histogram described in the previous section to incorporate activity weighting into the MinMax algorithm, whereby favoring colors with higher total activity weightings. Pseudocode for the proposed weighted MinMax algorithm is provided in Figure 2. The author's modifications to Xiang's best version of the MinMax algorithm given in Figure 1 are summarized below:

- (i) the centroid is selected as the representative color for the initial cluster in line 2, in line 4 the distance from each color,  $c$ , to its representative is weighted by  $H_w(c)$ , and
- (ii) the representative color of each cluster is set to its centroid at the end of each iteration of the primary for loop (lines 3-11) as in the LBG algorithm.

Gonzalez's theorem that the  $K$ -split produced by the MinMax algorithm has a maximum intercluster distance that is less than or equal to two times optimal solution does not apply to the weighted MinMax algorithm for the following reasons:

- (i) the weighted distance used in line 4 of Figure 2 does not satisfy the triangle inequality, and

<b>Input:</b>	$C$	The set of $N$ unique colors in $I$
	$K$	The numbers of colors to be selected.
		<i>Assumption: <math>N &gt; K</math>.</i>
	$H_w$	Activity weighted Histogram of $I$
<b>Output:</b>	$R_K$	Selected representatives for $I$
	$S_K$	$K$ -split for $C$
	// <b>Step 1:</b> Initialization	
1	$s_0 \leftarrow C = \{c_1, c_2, \dots, c_N\} \subseteq \text{RGB}$	
2	$r_0 \leftarrow \text{centroid of } C$	
	// <b>Step 2:</b> Iterative Creation of New Clusters	
3	<b>for</b> $k \leftarrow 1$ <b>upto</b> $K-1$	
4	$\delta \leftarrow \max \{d_s(c_i, r_j) \cdot H_w(c_i) \mid c_i \in s_j \text{ and } 0 \leq j \leq k-1\}$	
5	$c \leftarrow \text{one of the colors whose distance to its representative is } \delta$	
6	<b>move</b> $c$ to $s_k$	
7	$r_k \leftarrow c$	
	// <b>Step 3:</b> Reassignment of Colors to New Cluster	
8	<b>for each</b> $j \leftarrow 0$ <b>upto</b> $k-1$	
9	<b>for each</b> $c \in s_j$	
10	<b>if</b> $d_s(c, r_k) \leq d_s(c, r_j)$	
11	<b>move</b> $c$ to $s_k$	
12	<b>for each</b> $j \leftarrow 0$ <b>upto</b> $k$	
13	$r_j \leftarrow \text{centroid of } s_j$	
14	<b>return</b> $R_K$ and $S_K$	

Figure 2: **PSEUDOCODE FOR THE WEIGHTED MINMAX ALGORITHM.** The shaded lines indicate changes from the Xiang's version of the MinMax algorithm given in Figure 1.

(ii) the cluster representatives may change at the end of each iteration of for loop in lines 3-11 of Figure 2.

## 5. EXPERIMENTAL RESULTS

This section compares the effectiveness of the weighted MinMax algorithm to the MinMax algorithm in terms of both running time<sup>2</sup> and images quality. Twenty-five color images were used as an “in practice average-case” test set. The test set contains both natural and computer generated images, and contains a wide variety in both spatial and color resolution. In addition, the images in the test set exhibit many of the characteristics commonly found in color digital imagery:

- low frequency (smoothly varying) regions where false contours are more noticeable,

- isolated colors which force color shifting, and
- high frequency regions where sharpness may be deteriorated.

Image quality is difficult to quantify; however, the following distortion metrics are used: RMSE, WRMSE, MaxDiam, and MaxError. WRMSE weights the distance from each pixel in the quantized image to its corresponding pixel in the original image by the pixel's activity weighting. The diameter of a cluster produced by either the MinMax or weighted MinMax algorithm is defined to be the maximum distance between any two colors in the cluster. MaxDiam is defined to be the maximum diameter of all  $K$  clusters. MaxError is the maximum distance between any pixel in the quantized image and its corresponding pixel in the original image. All of these error metrics use the Euclidean distance function, not the scaled distance function,  $d_s$ .

<sup>2</sup> The algorithms were executed on a SUN Ultra 1 running Solaris 2.6 with 256 MB of RAM when the machine was otherwise idle.

#	<i>Time</i> (s)	<i>RMSE</i>	<i>WRMSE</i>	<i>MaxError</i>	<i>MaxDiam</i>
Max	198.35	11.86	10.34	36.40	93.11
Min	6.52	2.50	2.46	13.34	28.09
Avg	55.04	7.17	6.80	27.08	63.46
Max	285.88	12.01	10.37	96.65	176.78
Min	9.65	2.24	1.69	18.25	41.98
Avg	88.46	6.47	5.09	56.04	114.94

Table 1: **EMPIRICAL DATA FOR MINMAX AND WEIGHTED MINMAX.**  $K=256$ , 24 bits of precision. The weighted MinMax data is shaded.



Figure 3: **MINMAX (A-B) AND WEIGHTED MINMAX (C-D) QUANTIZATION OF SOLIDS AND LENA.**  $K=256$ , 24 bits of precision

Table 1 provides empirical data for both the MinMax and weighted MinMax algorithms. The average running time of the weighted MinMax algorithm is about 1.6 times that of the MinMax algorithm. However, Figure 3 illustrates that

the weighted MinMax algorithm produces higher quality quantized images than the MinMax algorithm. This is inconsistent with the observations that the MaxError and MaxDiam of the images quantized in Figure 3

using the weighted MinMax algorithm is larger than those quantized with the MinMax algorithm. Thus, neither the MaxError nor the MaxDiam metric is consistent with visual distortion. On the other hand, both the RMSE and the WRMSE of the images quantized in Figure 3 using the weighted MinMax algorithm is smaller than those quantized with the MinMax algorithm. Hence RMSE and WRMSE are more appropriate error measures than MaxError and MaxDiam are for color image quantization.

## 6. CONCLUSION

This paper conducted a comprehensive comparison between the MinMax algorithm and the proposed weighted MinMax algorithm. The comparison showed that MaxDiam and MaxError are not suitable distortion metrics for color image quantization. While the running time of the weighted MinMax algorithm is slower than the MinMax algorithm, the quality of quantized images produced was shown to be superior. A generalized method for applying activity weighting to any histogram-based color image quantization algorithm was also presented. The generalized activity weighting method was shown to be a fast and effective way to enhance the quality quantized images.

## REFERENCES

- [1] PAUL S. HECKBERT, Color Image Quantization for Frame Buffer Display, *Computer Graphics*, 16(3), July 1982, 297-304.
- [2] EVGENY ROYTMAN AND CRAIG GOTSMAN, Dynamic Color Quantization of Video Sequences, *IEEE Transactions on Visualization and Computer Graphics*, 1(3), September 1995, 274-86.
- [3] CHING-YUNG YANG AND JA-CHEN LIN, RWM-Cut for Color Image Quantization, *Computers and Graphics*, 20(4), 1996, 577-88.
- [4] TOLGA TASDIZEN, LALE AKARUN, AND CEM ERSOY, Color Quantization with Genetic Algorithms, *Signal Processing: Image Communication*, 12, 1998, 49-57.
- [5] PAULA J. REITAN, "Hybrid Approaches to Color Image Quantization." Ph.D. dissertation, University of Maryland Baltimore County, 1999.
- [6] TEOFILO F. GONZALEZ, Clustering to Minimize the Maximum Intercluster Distance, *Theoretical Computer Science*, 38(2-3), 1985, 23-306.
- [7] G. HOULE AND E. DUBOIS, Quantization of Color Images for Display on Graphics Terminals, *Proceedings IEEE Global Telecommunications Conference*, 2, Houston, Texas, (December 1-4, 1986), 1138-42.
- [8] NAFTALY GOLDBERG, Colour Image Quantization for High Resolution Graphics Display, *Image and Vision Computing*, , 1991, 303-12.
- [9] ALAIN TRÉMEAU, C. CHARRIER, AND H. CHERIFI, A Vector Quantization Algorithm based on the Nearest Neighbor of the Furthest Color, *Proceedings of International Conference on Image Processing*, 3, Santa Barbara, California, (October 26-29, 1997), 682-5.
- [10] ZHIGANG XIANG, Color Image Quantization by Minimizing the Maximum Intercluster Distance, *ACM Transactions on Graphics*, 16(3), July 1997, 260-76.
- [11] RAJA BALASUBRAMANIAN AND JAN P. ALLEBACH, A New Approach to Palette Selection for Color Images, *Human Vision, Visual Processing, and Digital Display II*, San Jose, California, (February 27-March 1, 1991), 58-69.
- [12] MICHAEL T. ORCHARD AND CHARLES A. BOUMAN, Color Quantization of Images, *IEEE Transactions on Acoustics, Speech and Signal Processing*, 39(12), December 1991, 2677-90.
- [13] RAJA BALASUBRAMANIAN, JAN P. ALLEBACH, AND CHARLES A. BOUMAN, Color-Image Quantization with use of a Fast Binary Splitting Technique, *Journal of the Optical Society of America*, 11(11), November 1994, 2777-86.
- [14] NAVIN CHADDHA, WEE-CHIEW TAN, AND TERESA H.Y. MENG, Color Quantization of Images Based on Human Visual Perception, *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, V, Adelaide, South Australia, (April 19-22, 1994), 89-92.
- [15] KYEONG-MAN KIM, CHAE-SOO LEE, EUNG-JOO LEE, AND YEONG-HO HA, Color Image Quantization using Weighted Distortion Measure of HVS Color Activity, *Proceedings of 3rd IEEE International Conference on Image Processing*, 3, Lausanne, Switzerland, (September 16-19, 1996), 1035-9.
- [16] KYEONG-MAN KIM, CHAE-SOO LEE, EUNG-JOO LEE, AND YEONG-HO HA, Color Image Quantization and Dithering Method Based on Human Visual System Characteristics, *The Journal of Imaging Science and Technology*, 40(6), November/December 1996, 502-9.
- [17] YANG-WOO PARK, EUNG-JOO LEE, KYEONG-MAN KIM, CHE-SOO LEE, AND YEONG-HO HA, Color Enhancement by New Quantization of Fast Clustering Method, *Proceedings of Imaging Science and Technology: Evolution and Promise*, Chiba, Japan, (September 11-14, 1996), 275-81.